

Baryon number violation due to brane dynamics

Tomoko Uesugi^(1,2), Shunsuke Fujii⁽³⁾ and Tetsuya Shiromizu^(3,4,5)

⁽¹⁾*Institute of Physics, Meiji Gakuin University, Yokohama 244-8539, Japan*

⁽²⁾*Department of Physics, Graduate School of Humanities and Sciences, Ochanomizu University, Tokyo 112-8610, Japan*

⁽³⁾*Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan*

⁽⁴⁾*Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan and*

⁽⁵⁾*Advanced Research Institute for Science and Engineering, Waseda University, Tokyo 169-8555, Japan*

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We derive the low energy effective theory for two brane system with bulk complex scalar fields. Then we find that higher order corrections to the effective equation for scalar fields on the brane lead us to the baryon number(B) violating process if the complex scalar field is regarded as a particle carrying the baryon number current. We see that the motion of the brane leads us the baryon number violation via the current-curvature coupling even if the potential for scalar fields do not exist. We also estimate the net baryon number assuming a C/CP violating interaction.

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I. INTRODUCTION

According to current various observations, our universe is dominated by matters. We do not observe a large amount of anti-matters. Since we have no reason for the domination of the matters at the beginning of our universe, the origin of this baryon-antibaryon asymmetry is one of important problems in cosmology (See Ref. [1] for the recent review and references). On the other hand, the recent progress of superstring theory provides us the new picture of our universe, braneworld. Therein our universe is described by the motion of thin wall in higher dimensional spacetimes (See Ref. [2] for the review). In general the effect of the extra dimension is important in the very early universe. Since baryogenesis works in the very early universe, it is natural to ask if the new type of the realisation of baryogenesis is possible in the braneworld context. Actually, a couple of ideas have been proposed [3, 4, 5].

The end of this paper is reexamination of a baryon number violating process discussed in the braneworld context [3]. In Ref. [3], a scenario of baryogenesis was proposed in Randall-Sundrum type single brane model [6] with bulk complex scalar fields. The bulk complex scalar field may be regarded as a particle carrying the baryon number current like squarks. Therein the effective action on the brane was derived by using the adS/CFT correspondence [7], and it was shown that the baryon number violating curvature-current interactions is included in the Kaluza-Klein corrections. The mechanism is shared with other works in the braneworld [8] where the escape of massive scalar fields and charge via Kaluza-Klein modes were discussed. However, the adS/CFT correspondence leaves us an ambiguity in braneworld because it has not been yet established completely and “CFT” effective action cannot be determined fully. In this paper, therefore, we carefully derive the effective equations for Randall-Sundrum two brane system [9] with the bulk complex scalar field in order to obtain a definite answer for the effective equation. We use the long wave approximation

[10] for the derivation. In the case of two branes the adS/CFT correspondence does not hold. Rather say, we can solve the bulk field equations under the boundary junction condition without any ambiguities.

The rest of this paper is organized as follows. In Sec. II we describe our models. Then, in Sec. III, we solve the bulk equation to derive the effective equation on branes. Since we focus on baryogenesis, we derive the effective equation for the complex scalar field by using the long wave approximation. For simplicity, we will assume that contributions from the radion and the scalar field to the spacetime dynamics on the brane are negligible. Moreover, we assume that the contribution to the correction is dominated by curvature-complex scalar field interactions. Using effective equations, in Sec. IV, we will discuss the non-conservation of the baryon number current and evaluate the net baryon number to entropy ratio by supposing plausible CP violating interaction. Finally we give a summary in Sec. V.

II. MODEL

We consider Z_2 symmetric Randall-Sundrum type two brane system with the bulk complex scalar field. The bulk and brane actions are given by

$$S_{\text{bulk}} = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} ({}^{(5)}R - 2\Lambda) - \int d^5x \sqrt{-g} \left[\frac{1}{2} g^{MN} \nabla_M \Phi \nabla_N \Phi^* + V(|\Phi|) \right] \quad (1)$$

and

$$S_{\text{brane}}^{(\pm)} = \int d^4x \sqrt{-g_{(\pm)}} \left[-\sigma_{\pm}(|\phi_{\pm}|) + \mathcal{L}_{\pm} \right]. \quad (2)$$

g_{MN} , ${}^{(5)}R$, Λ and Φ are bulk metric, five dimensional Ricci scalar, bulk cosmological constant and the $U(1)$ bulk complex scalar field, respectively. $g_{(\pm)\mu\nu}$ are the

brane induced metric and ϕ_{\pm} are complex scalar fields Φ on each brane. \mathcal{L}_{\pm} are the Lagrangian densities for matter fields localized on branes. Later we will suppose that the spacetime dynamics on the brane is mainly supported by matters and the contribution from the bulk complex scalar field to the spacetime dynamics is negligible. The bulk stress tensor is

$$T_{MN} = \frac{1}{2}(\nabla_M \Phi \nabla_N \Phi^* + \nabla_M \Phi^* \nabla_N \Phi) - g_{MN} \left(\frac{1}{2} |\nabla \Phi|^2 + V(|\Phi|) \right) - \kappa^{-2} \Lambda g_{MN}. \quad (3)$$

III. EFFECTIVE EQUATION FOR SCALAR FIELDS ON BRANES

In this section, we derive the effective equation for scalar fields on branes using the long wave approximation [10]. We first write down equations for present system and then solve them iteratively up to the second order. We would stress that we focus on the scalar field Φ because we are interested in the mechanism of baryogenesis.

We take the metric ansatz

$$ds^2 = e^{2\varphi(x)} dy^2 + g_{\mu\nu}(y, x) dx^\mu dx^\nu \quad (4)$$

and employ (1+4) decomposition. Without a loss of generality, we can suppose that branes are located in $y = y_+ = 0$ and $y = y_- = y_0 > 0$. Then we expand the metric $g_{\mu\nu}$, the extrinsic curvature $K_{\mu\nu} = \frac{1}{2} e^{-\varphi} \partial_y g_{\mu\nu}$ and Φ in terms of the small dimensionless parameter ϵ :

$$g_{\mu\nu} = a^2(y, x) (h_{\mu\nu} + g_{\mu\nu}^{(1)} + \dots) \quad (5)$$

$$K_{\mu\nu}^{(0)} = K_{\mu\nu}^{(0)} + K_{\mu\nu}^{(1)} + \dots \quad (6)$$

$$\Phi = \Phi^{(0)} + \Phi^{(1)} + \dots \quad (7)$$

Here ϵ is the square of the ratio of the bulk curvature scale ℓ to the brane intrinsic curvature scale L :

$$\epsilon = \left(\frac{\ell}{L} \right)^2. \quad (8)$$

A. Basic equations

The “evolutional” equation is given by

$$e^{-\varphi} \partial_y \tilde{K}_{\nu}^{\mu} = {}^{(4)}\tilde{R}_{\nu}^{\mu} - \kappa^2 \left({}^{(5)}T_{\nu}^{\mu} - \frac{1}{4} \delta_{\nu}^{\mu} {}^{(5)}T_{\alpha}^{\alpha} \right) - K \tilde{K}_{\nu}^{\mu} - e^{-\varphi} (D^{\mu} D_{\nu} e^{\varphi})_{\text{traceless}}, \quad (9)$$

where \tilde{K}_{ν}^{μ} is the traceless part of the extrinsic curvature K_{ν}^{μ} , ${}^{(4)}\tilde{R}_{\nu}^{\mu}(g)$ is the four dimensional Ricci tensor ${}^{(4)}R_{\nu}^{\mu}(g)$, and D_{μ} is the covariant derivative with respect

to $g_{\mu\nu}$. $(\dots)_{\text{traceless}}$ stands for the traceless part of (\dots) . The equation for the trace part, K , is

$$e^{-\varphi} \partial_y K = {}^{(4)}R(g) - \kappa^2 \left(T_{\mu}^{\mu} - \frac{4}{3} T_M^M \right) - K^2 - e^{-\varphi} D^2 e^{\varphi}. \quad (10)$$

The constraint equations are

$$-\frac{1}{2} \left[{}^{(4)}R - \frac{3}{4} K^2 + \tilde{K}_{\nu}^{\mu} \tilde{K}_{\mu}^{\nu} \right] = \kappa^2 {}^{(5)}T_{yy} e^{-2\varphi}, \quad (11)$$

and

$$D_{\nu} K_{\mu}^{\nu} - D_{\mu} K = \kappa^2 {}^{(5)}T_{\mu y} e^{-\varphi}. \quad (12)$$

The equation for the scalar field is

$$e^{-2\varphi} \partial_y^2 \Phi_i + D^{\mu} \varphi D_{\mu} \Phi_i + K e^{-\varphi} \partial_y \Phi_i + D^2 \Phi_i - \partial_{\Phi_i} V = 0 \quad (13)$$

where $i = 1, 2$ and $\Phi = \Phi_1 + i\Phi_2$. $\Phi_i (i = 1, 2)$ is the real and the pure imaginary component of the complex scalar field Φ .

By virtue of Z_2 symmetry, the junction condition on each brane are given by

$$(K_{\nu}^{\mu} - \delta_{\nu}^{\mu} K)|_{y=y_{\pm}} = \frac{\kappa^2}{2} \left(\pm \sigma_{\pm} \delta_{\nu}^{\mu} \mp T_{\nu}^{\mu(\pm)} \right) \quad (14)$$

and

$$e^{-\varphi} \partial_y \Phi_i|_{y=y_{\pm}} = \pm \frac{1}{2} \tilde{\sigma}_{\pm}'(\phi_i^{\pm}), \quad (15)$$

where $\phi_i^{\pm} = \Phi_i(y = y_{\pm})$, $\sigma_{\pm}(\phi_{\pm}) = \pm \sigma_0 + \tilde{\sigma}_{\pm}(\phi_{\pm})$ and $\tilde{\sigma}_{\pm}'(\phi_{\pm}) = \lim_{y \rightarrow y_{\pm}} \partial_{\Phi_i} \tilde{\sigma}_{\pm}(\Phi_i)$. We set σ_0 to be positive. Then the brane at $y = 0 (y = y_0)$ has the positive(negative) tension. The junction condition corresponds to the boundary condition.

B. Background

The evolutional equation of \tilde{K}_{ν}^{μ} for the background spacetime is

$$e^{-\varphi} \partial_y \tilde{K}_{\nu}^{\mu(0)} = - \tilde{K}^{(0)} \tilde{K}_{\nu}^{\mu(0)}. \quad (16)$$

The constraint equations are

$$-\frac{1}{2} \left[-\frac{3}{4} \tilde{K}^{(0)2} + \tilde{K}_{\nu}^{\mu(0)} \tilde{K}_{\mu}^{\nu(0)} \right] = \kappa^2 {}^{(0)}T_{yy} e^{-2\varphi} \quad (17)$$

and

$$D_{\nu} \tilde{K}_{\mu}^{\nu(0)} - D_{\mu} \tilde{K}^{(0)} = 0. \quad (18)$$

The equation for the scalar field becomes

$$e^{-2\varphi} \partial_y^2 \Phi_i^{(0)} + \tilde{K}^{(0)} e^{-\varphi} \partial_y \Phi_i^{(0)} = 0. \quad (19)$$

The junction conditions are

$$(\overset{(0)}{K}_{\mu\nu} - g_{\mu\nu} \overset{(0)}{K})_{y=y_{(\pm)}} = \pm \frac{\kappa^2}{2} \sigma_0 g_{\mu\nu} \quad (20)$$

and

$$\partial_y \overset{(0)}{\Phi}_i|_{y=y_{\pm}} = 0. \quad (21)$$

After all, we can easily find the solution for the extrinsic curvature as

$$\overset{(0)}{K}_{\nu}^{\mu} = -\frac{1}{\ell} \delta_{\nu}^{\mu} \quad (22)$$

where $\ell^{-1} = \frac{1}{6} \kappa^2 \sigma_0$. Then the background spacetime is determined as

$$g_{\mu\nu}^{(0)} = a^2(x, y) h_{\mu\nu}(x) \quad (23)$$

where

$$a(x, y) = e^{-\frac{d(x, y)}{\ell}} \quad (24)$$

and $d = \int_0^y dy e^{\varphi(x)} = y e^{\varphi(x)}$. $h_{\mu\nu}(x)$ is the induced metric on the positive tension brane at $y = 0$. The zero-th order solution of scalar fields is a function only of x ,

$$\overset{(0)}{\Phi}_i(y, x) = \eta_i(x). \quad (25)$$

C. First order

The evolutionary equation of \tilde{K}_{ν}^{μ} in the first order is

$$e^{-\varphi} \partial_y \overset{(1)}{\tilde{K}}_{\nu}^{\mu} = -\kappa^2 \left(T_{\nu}^{\mu} - \frac{1}{4} \delta_{\nu}^{\mu} T \right)^{(1)} - \overset{(0)}{K} \overset{(1)}{\tilde{K}}_{\nu}^{\mu} + \overset{(4)}{R}_{\nu}^{\mu} \tilde{R}_{\nu}^{\mu} \quad (26)$$

$$- \frac{1}{a^2} (e^{-\varphi} \mathcal{D}^{\mu} \mathcal{D}_{\nu} e^{\varphi})_{\text{traceless}},$$

where \mathcal{D}_{μ} is the covariant derivative with respect to the metric $h_{\mu\nu}$. $(\dots)^{(1)}$ stands for the first order part of (\dots) . Then the solution is given by

$$\begin{aligned} \overset{(1)}{\tilde{K}}_{\nu}^{\mu}(y, x) &= -\frac{\ell}{2} a^{-2(4)} \tilde{R}_{\nu}^{\mu}(h) \\ &\quad - a^{-2} \left(\mathcal{D}^{\mu} \mathcal{D}_{\nu} d + \frac{1}{\ell} \mathcal{D}^{\mu} d \mathcal{D}_{\nu} d \right)_{\text{traceless}} \\ &\quad + \frac{\kappa^2 \ell}{4} a^{-2} (\mathcal{D}^{\mu} \eta \mathcal{D}_{\nu} \eta^{*} + \mathcal{D}^{\mu} \eta^{*} \mathcal{D}_{\nu} \eta)_{\text{traceless}} \\ &\quad + \chi_{\nu}^{\mu}(x) a^{-4}, \end{aligned} \quad (27)$$

where $\overset{(4)}{R}_{\mu\nu}(h)$ is the Ricci scalar of the metric $h_{\mu\nu}$, $\chi_{\nu}^{\mu}(x)$ is the “constant” of integration, and $\eta = \eta_1 + i\eta_2$. η_1 and η_2 are real and pure imaginary parts of η . The trace part of the extrinsic curvature is directly computed from the Hamiltonian constraint and the result is

$$\begin{aligned} \overset{(1)}{K}(y, x) &= -\frac{\ell}{6a^2} \overset{(4)}{R}(h) - \frac{1}{a^2} \left(\mathcal{D}^2 d - \frac{1}{\ell} (\mathcal{D}d)^2 \right) \\ &\quad + \frac{\kappa^2 \ell}{3} \frac{1}{a^2} \left(\frac{1}{2} |\mathcal{D}\eta|^2 + a^2 V(\eta_i) \right). \end{aligned} \quad (28)$$

We can summarise the result as

$$\begin{aligned} \overset{(1)}{K}_{\nu}^{\mu} - \delta_{\nu}^{\mu} \overset{(1)}{K} &= -\frac{\ell}{2} a^{-2(4)} G_{\nu}^{\mu}(h) - a^{-2} \left[\mathcal{D}^{\mu} \mathcal{D}_{\nu} d - \delta_{\nu}^{\mu} \mathcal{D}^2 d \right. \\ &\quad \left. + \frac{1}{\ell} \left(\mathcal{D}^{\mu} d \mathcal{D}_{\nu} d + \frac{1}{2} \delta_{\nu}^{\mu} (\mathcal{D}d)^2 \right) \right] \\ &\quad + \frac{\kappa^2 \ell}{4} a^{-2} \left[\mathcal{D}^{\mu} \eta \mathcal{D}_{\nu} \eta^{*} + \mathcal{D}^{\mu} \eta^{*} \mathcal{D}_{\nu} \eta \right. \\ &\quad \left. - \delta_{\nu}^{\mu} (|\mathcal{D}\eta|^2 + a^2 V(\eta)) \right] + \chi_{\nu}^{\mu}(x) a^{-4}. \end{aligned} \quad (29)$$

Applying the junction condition on the positive tension brane to the above equation, we obtain

$$\begin{aligned} 0 &= -\frac{\kappa^2}{2} (\tilde{\sigma} + \delta_{\nu}^{\mu} - \overset{(+)}{T}_{\nu}^{\mu}) - \frac{\ell}{2} \overset{(4)}{G}_{\nu}^{\mu}(h) \\ &\quad + \frac{\kappa^2 \ell}{4} \left[\mathcal{D}^{\mu} \eta \mathcal{D}_{\nu} \eta^{*} + \mathcal{D}^{\mu} \eta^{*} \mathcal{D}_{\nu} \eta - \delta_{\nu}^{\mu} (|\mathcal{D}\eta|^2 + V(\eta)) \right] \\ &\quad + \chi_{\nu}^{\mu}(x). \end{aligned} \quad (30)$$

From the junction condition on the negative tension brane, we also have

$$\begin{aligned} 0 &= \frac{\kappa^2}{2} (\tilde{\sigma} - \delta_{\nu}^{\mu} - a_0^{-2} \overset{(-)}{T}_{\nu}^{\mu}) - \frac{\ell}{2} a_0^{-2(4)} G_{\nu}^{\mu}(h) \\ &\quad - a_0^{-2} \left[\mathcal{D}^{\mu} \mathcal{D}_{\nu} d_0 - \delta_{\nu}^{\mu} \mathcal{D}^2 d_0 \right. \\ &\quad \left. + \frac{1}{\ell} \left(\mathcal{D}^{\mu} d_0 \mathcal{D}_{\nu} d_0 + \frac{1}{2} \delta_{\nu}^{\mu} (\mathcal{D}d_0)^2 \right) \right] \\ &\quad + \frac{\kappa^2 \ell}{4} a_0^{-2} \left[\mathcal{D}^{\mu} \eta \mathcal{D}_{\nu} \eta^{*} + \mathcal{D}^{\mu} \eta^{*} \mathcal{D}_{\nu} \eta \right. \\ &\quad \left. - \delta_{\nu}^{\mu} (|\mathcal{D}\eta|^2 + a_0^2 V(\eta)) \right] + \chi_{\nu}^{\mu}(x) a_0^{-4}. \end{aligned} \quad (31)$$

Then, eliminating the constant of integration $\chi_{\mu\nu}(x)$ from the above two equations, we can obtain the effective gravitational equation on the positive tension brane

$$\begin{aligned} &(1 - a_0^2) \overset{(4)}{G}_{\mu\nu}(h) \\ &= \frac{2}{\ell} a_0^2 \left[\mathcal{D}^{\mu} \mathcal{D}_{\nu} d_0 - \delta_{\nu}^{\mu} \mathcal{D}^2 d_0 + \frac{1}{\ell} \left(\mathcal{D}^{\mu} d_0 \mathcal{D}_{\nu} d_0 \right. \right. \\ &\quad \left. \left. + \frac{1}{2} \delta_{\nu}^{\mu} (\mathcal{D}d_0)^2 \right) \right] + \kappa^2 \ell^{-1} \left(\overset{(+)}{T}_{\mu\nu} + a_0^2 \overset{(-)}{T}_{\mu\nu} \right) \\ &\quad + \frac{\kappa^2}{2} (1 - a_0^2) \left[\mathcal{D}^{\mu} \eta \mathcal{D}_{\nu} \eta^{*} + \mathcal{D}^{\mu} \eta^{*} \mathcal{D}_{\nu} \eta - \delta_{\nu}^{\mu} (|\mathcal{D}\eta|^2 \right. \\ &\quad \left. + (1 + a_0^2) V(\eta)) \right] - \frac{\kappa^2}{\ell} (\tilde{\sigma}^{+} + a_0^4 \tilde{\sigma}^{-}) h_{\mu\nu}. \end{aligned} \quad (32)$$

This is not our main conclusion. What we want to do in this section is the derivation of the effective equation for scalar fields.

Since we are interested in the interaction between the curvature and scalar fields, we omit contributions of the radion field, that is, the derivative of φ , $D_{\mu}\varphi$. Then the bulk equation in the first order is

$$e^{-2\varphi} \partial_y^2 \overset{(1)}{\Phi}_i + \overset{(0)}{K} e^{-\varphi} \partial_y \overset{(1)}{\Phi}_i + (D^2 \Phi_i)^{(1)} - \partial_{\eta_i} V(\eta) = 0 \quad (33)$$

By performing the integration over y , the above equation becomes

$$\partial_y \Phi_i^{(1)} = \frac{\ell}{2} \left(a^{-2} \mathcal{D}^2 \eta_i - \frac{1}{2} \partial_{\eta_i} V(\eta) \right) e^\varphi + \frac{\chi_i(x)}{a^4}, \quad (34)$$

where $\chi_i(x)$ is the integral constant. We assume $\Phi_i^{(1)}(0, x) = 0$. Applying the junction condition for scalar fields to the above equation, we obtain two equations

$$\frac{1}{2} \tilde{\sigma}'_{+i} = \frac{\ell}{2} \left(\mathcal{D}^2 \eta_i - \frac{1}{2} \partial_{\eta_i} V(\eta) \right) + \chi_i(x) e^{-\varphi} \quad (35)$$

and

$$-\frac{1}{2} \tilde{\sigma}'_{-i} = \frac{\ell}{2} \left(a_0^{-2} \mathcal{D}^2 \eta_i - \frac{1}{2} \partial_{\eta_i} V(\eta) \right) + \chi_i(x) e^{-\varphi} a_0^{-4} \quad (36)$$

Then, eliminating the integral constant $\chi_i(x)$ from the above two equations, we obtain the effective equation for scalar fields in the first order

$$(1 - a_0^2) \mathcal{D}^2 \eta_i - \frac{1}{2} (1 - a_0^4) \partial_{\eta_i} V(\eta) = \frac{1}{\ell} (\tilde{\sigma}'_{+i} + a_0^4 \tilde{\sigma}'_{-i}) \quad (37)$$

It is easy to see that the baryon number current is conserved up to this first order. Therefore we will consider the second order corrections into the effective equation for scalar fields.

D. Second order

From now on we assume that the contribution from η_i to the effective theory is negligible compared to that from ${}^{(4)}R(D\eta)^2$ and $T_{\mu\nu}^{(+)}(D\eta)^2$ in the calculation of the second order. This is because we are interested in the interaction between scalar fields and the brane intrinsic curvature.

The equation for scalar fields in the second order is

$$e^{-2\varphi} \partial_y^2 \Phi_i^{(2)} + K e^{-\varphi} \partial_y \Phi_i^{(2)} + K e^{-\varphi} \partial_y \Phi_i^{(1)} + (D^2 \Phi_i)^{(2)} = 0. \quad (38)$$

Then the formal integration of Eq. (38) over y gives us

$$\partial_y \Phi_i^{(2)} = -a^{-4} \int^y dy e^{2\varphi} a^4 \left(K e^{-\varphi} \partial_y \Phi_i^{(1)} + (D^2 \Phi_i)^{(2)} \right) + a^{-4} \chi_i^{(2)}(x). \quad (39)$$

To proceed the calculation further, we need to determine $g_{\mu\nu}^{(1)}$ in

$$g_{\mu\nu} = a^2 (h_{\mu\nu} + g_{\mu\nu}^{(1)}). \quad (40)$$

Assuming $g_{\mu\nu}^{(1)} = 0$ at $y = y_+ = 0$ and using the results in the previous section, we obtain

$$g_{\mu\nu}^{(1)} = -\frac{\ell^2}{2} (a^{-2} - 1) \left({}^{(4)}R_{\mu\nu}(h) - \frac{1}{6} h_{\mu\nu} {}^{(4)}R(h) \right)$$

$$\begin{aligned} & + \frac{\kappa^2}{4} \ell^2 (a^{-2} - 1) \left(\mathcal{D}_\mu \eta \mathcal{D}_\nu \eta^* + \mathcal{D}_\mu \eta^* \mathcal{D}_\nu \eta \right. \\ & \left. - \frac{1}{3} h_{\mu\nu} |\mathcal{D}\eta|^2 \right) + \frac{\kappa^2}{6} \ell V(\eta) h_{\mu\nu} d \\ & + \frac{\ell}{2} (a^{-4} - 1) \chi_{\mu\nu}(x), \end{aligned} \quad (41)$$

where

$$\begin{aligned} \chi_{\mu\nu}(x) &= \frac{\ell}{2} ({}^{(4)}G_{\mu\nu}(h) - \kappa^2 \ell^{-1} T_{\mu\nu}^{(+)} + \frac{\kappa^2}{2} \tilde{\sigma}_+ (\phi_+) h_{\mu\nu} \\ & - \frac{\kappa^2 \ell}{4} \left[\mathcal{D}_\mu \eta \mathcal{D}_\nu \eta^* + \mathcal{D}_\mu \eta^* \mathcal{D}_\nu \eta \right. \\ & \left. - h_{\mu\nu} \left((\mathcal{D}\eta)^2 + V(\eta) \right) \right]. \end{aligned} \quad (42)$$

Then we can calculate $(D^2 \Phi_i)^{(2)}$ as

$$\begin{aligned} (D^2 \Phi_i)^{(2)} &= a^{-2} \mathcal{D}^2 \Phi_i^{(1)} - a^{-4} \frac{h^{\alpha\beta}}{2} g_{\alpha\beta}^{(1)} \mathcal{D}_\mu (a^2 \mathcal{D}^\mu \Phi_i^{(0)}) \\ & + a^{-4} \mathcal{D}_\mu \left[a^2 \left(-g^{\mu\nu} + \frac{h^{\alpha\beta}}{2} g_{\alpha\beta}^{(1)} h^{\mu\nu} \right) \mathcal{D}_\nu \Phi_i^{(0)} \right] \\ & = a^{-2} \mathcal{D}^2 \Phi_i^{(1)} + \frac{\ell^2}{12} a^{-2} (a^{-2} - 1) {}^{(4)}R(h) \mathcal{D}^2 \eta_i \\ & + a^{-2} \mathcal{D}_\mu \left[\left\{ \frac{\ell^2}{2} (a^{-2} - 1) ({}^{(4)}R^{\mu\nu}(h) \right. \right. \\ & \left. \left. - \frac{1}{3} h^{\mu\nu} {}^{(4)}R(h) \right) \right. \\ & \left. \left. - \frac{\ell}{2} (a^{-4} - 1) \chi^{\mu\nu} \right\} \mathcal{D}_\nu \eta_i \right] \end{aligned} \quad (43)$$

Substituting this for Eq. (39), we see

$$\begin{aligned} e^{-\varphi} \partial_y \Phi_i^{(2)} &= -\mathcal{D}^2 \left[\frac{\ell^2}{16} (a^{-6} + a^{-2}) \tilde{\sigma}'_{+i} \right. \\ & + \frac{\ell^3}{4} \left\{ a^{-4} \left(\frac{d}{\ell} + \frac{1}{2} a^2 \right) - \frac{1}{4} (a^{-6} + a^{-2}) \right\} \mathcal{D}^2 \eta_i \\ & + \frac{\ell^3}{8} \left\{ a^{-2} \left(\frac{d}{\ell} + \frac{1}{2} \right) + \frac{1}{4} (a^{-6} + a^{-2}) \right\} \partial_{\eta_i} V(\eta) \Big] \\ & + \frac{\ell^3}{48} a^{-2(4)} R(h) \partial_{\eta_i} V(\eta) \\ & + \frac{\ell^2}{12} a^{-6} \chi_i e^{-\varphi(4)} R(h) - \frac{\ell^3}{24} a^{-2(4)} R \mathcal{D}^2 \eta_i \\ & + \mathcal{D}_\mu \left[\left\{ -\frac{\ell^3}{2} \left(\frac{d}{\ell} a^{-4} + \frac{1}{2} a^{-2} \right) ({}^{(4)}R^{\mu\nu}(h) \right. \right. \\ & \left. \left. - \frac{1}{3} h^{\mu\nu} {}^{(4)}R(h) \right) + \frac{\ell^2}{4} (a^{-6} + a^{-2}) \chi^{\mu\nu} \right\} \mathcal{D}_\nu \eta_i \Big] \\ & + \chi_i^{(2)} a^{-4} e^{-\varphi}. \end{aligned} \quad (44)$$

In the above we used the solution to $\Phi_i^{(1)}$

$$\Phi_i^{(1)}(y, x) = \frac{\ell}{8} (a^{-4} - 1) \tilde{\sigma}'_{+i}$$

$$\begin{aligned}
& + \frac{\ell^2}{4} \left[(a^{-2} - 1) - \frac{1}{2}(a^{-4} - 1) \right] \mathcal{D}^2 \eta_i \\
& - \frac{\ell^2}{4} \left[\frac{d}{\ell} - \frac{1}{4}(a^{-4} - 1) \right] \partial_{\eta_i} V. \quad (45)
\end{aligned}$$

Using the junction condition and following the same argument as the previous section, we obtain the effective equation for the scalar field up to the second order,

$$\begin{aligned}
& (1 - a_0^2) \mathcal{D}^2 \eta_i - \frac{1}{2} (1 - a_0^4) \partial_{\eta_i} V(\eta_i) \\
& = \ell^{-1} (\tilde{\sigma}'_{+i} + a_0^4 \tilde{\sigma}'_{-i}) - \frac{\ell^2}{24} (1 - a_0^2)^{(4)} R(h) \partial_{\eta_i} V(\eta) \\
& - \frac{\ell}{6} (1 - a_0^{-2}) \chi_i e^{-\varphi(4)} R(h) + \frac{\ell^2}{12} (1 - a_0^2)^{(4)} R(h) \mathcal{D}^2 \eta_i \\
& + \mathcal{D}_\mu \left[\left\{ \ell^2 \left(-\frac{d_0}{\ell} + \frac{1}{2} (1 - a_0^2) \right) \left({}^{(4)} R^{\mu\nu}(h) \right. \right. \right. \\
& \left. \left. \left. - \frac{1}{3} h^{\mu\nu(4)} R(h) \right) - \frac{\ell}{2} (2 - a_0^{-2} - a_0^2) \chi^{\mu\nu} \right\} \mathcal{D}_\nu \eta_i \right] \\
& + \mathcal{D}^2 \left[\frac{\ell}{8} (2 - a_0^{-2} - a_0^2) \tilde{\sigma}'_{+i} + \frac{\ell^2}{2} \left\{ -\frac{d_0}{\ell} + \frac{1}{2} (1 - a_0^2) \right. \right. \\
& \left. \left. - \frac{1}{4} (2 - a_0^{-2} - a_0^2) \right\} \mathcal{D}^2 \eta_i + \frac{\ell^2}{4} \left\{ -\frac{d_0}{\ell} a_0^2 \right. \right. \\
& \left. \left. + \frac{1}{2} (1 - a_0^2) + \frac{1}{4} (2 - a_0^{-2} - a_0^2) \right\} \partial_{\eta_i} V \right] \quad (46)
\end{aligned}$$

Now we can see the coupling between the brane intrinsic curvature and the scalar field. As seen in the next section, this gives us the possibility of the baryon number violation due to the spacetime dynamics.

IV. BARYOGENESIS

In this section we discuss the baryon number violation in the Randall-Sundrum braneworld context. The point is as follows. Since the bulk complex scalar field has the global U(1) symmetry in five dimensions, there is the current $J^M = i(\Phi \nabla^M \Phi^* - \Phi^* \nabla^M \Phi)$ satisfying the local conservation law $\nabla_M J^M = 0$. However, this conservation does not mean the conservation of the projected current J^μ which is the observed quantity on the brane. Actually, we see $D_\mu J^\mu \sim -\partial_y J^y$. Depending on the value of $\partial_y J^y$, the projected current is not conserved in general.

Let us define the the baryon number current by

$$\begin{aligned}
J_\mu & := i(\Phi \partial_\mu \Phi^* - \Phi^* \partial_\mu \Phi)|_{y=y_+} \\
& = 2i(\eta_1 \partial_\mu \eta_2 - \eta_2 \partial_\mu \eta_1). \quad (47)
\end{aligned}$$

Then the divergence of the current becomes

$$\mathcal{D}_\mu J^\mu = 2(\eta_1 \mathcal{D}^2 \eta_2 - \eta_2 \mathcal{D}^2 \eta_1). \quad (48)$$

This is the equation on the positive tension brane. The right-hand side of this equation can be evaluated using

the field equation. Since we are supposing that the space-time dynamics is mainly governed by localized matters with $T_{\mu\nu}^{(\pm)}$ on the brane, we can see that Eq. (48) becomes

$$\begin{aligned}
\mathcal{D}_\mu J^\mu & \simeq \ell^2 \mathcal{D}_\mu \left[\left\{ \left(\frac{-\frac{d_0}{\ell}}{1 - a_0^2} + \frac{1}{2} \right) \left({}^{(4)} R^\mu_\nu(h) - \frac{1}{3} \delta^\mu_\nu {}^{(4)} R \right) \right. \right. \\
& \left. \left. + \frac{1}{4} (a_0^{-2} - 1) \left({}^{(4)} G^\mu_\nu(h) - \kappa^2 \ell^{-1} T_{\nu}^{(+)\mu} \right) \right. \right. \\
& \left. \left. - \frac{\kappa^2}{2} (\mathcal{D}^\mu \eta \mathcal{D}_\nu \eta^* + \mathcal{D}^\mu \eta^* \mathcal{D}_\nu \eta - \delta^\mu_\nu |\mathcal{D} \eta|^2) \right\} J^\nu \right] \quad (49)
\end{aligned}$$

Therefore the current J^μ is not conserved in general. Now we consider the radiation dominated universe.

From Eq. (49) we obtain the equation for the total charge $Q_B = \int d^3 \mathbf{x} j^0$

$$\dot{Q}_B \sim \ell^2 {}^{(4)} \dot{R} Q_B. \quad (50)$$

In the braneworld context, \dot{R} is written as [11]

$${}^{(4)} \dot{R} = -3(1+w)H \left[\frac{1-3w}{M_4^2} - \frac{(1+3w)\rho}{3M_5^6} \right] \rho, \quad (51)$$

where $w = P/\rho$ and H is the Hubble parameter. In the radiation dominated era, ${}^{(4)} \dot{R} \sim HT^8 M_5^{-6}$. Then it is easy to see that the rate of the baryon number violating process is given by

$$\Gamma_B \sim \ell^2 {}^{(4)} \dot{R} \sim \frac{\ell^2 HT^8}{M_5^6} \sim \frac{M_4^4 HT^8}{M_5^{12}}. \quad (52)$$

In the above we used $\ell \sim M_4^2/M_5^3$. Now we can estimate the decoupling temperature T_D and then $T_D \sim M_5^{3/2}/M_4^{1/2}$. We suppose that J^μ is a current which produces net $B - L$. Then the finite baryon number density will be left after the electroweak sphaleron process occurs [12]. The net baryon number density depends on the source of CP-violation. For example, we consider the CP violating interaction $\mathcal{L}_{\text{int}} \sim f M_*^{-2} \partial_\mu {}^{(4)} R J^\mu$ [13] which may be originated from the non-perturbative effect of the quantum gravity or the string theory (See Ref. [14] for another proposal.). M_* is usually expected to be an corresponding scale to the bulk curvature, ℓ^{-1} . f is a factor which may be expected to be a small number. This interaction is the same as a CP violating interaction supposed in the spontaneous baryogenesis [15]. Actually, we can produce this interaction \mathcal{L}_{int} from the CP violating interaction term assumed in the usual spontaneous baryogenesis [3]. Interestingly, this interaction breaks the CPT spontaneously due to the dynamics of the expanding universe. Therefore, in this case, the out of thermal equilibrium is not necessary for the baryogenesis. According to the same argument as Ref. [3], the produced baryon to entropy ratio is [16]

$$n_B/s \sim f {}^{(4)} \dot{R} / M_*^2 T|_{T=T_D}$$

$$\begin{aligned} &\sim f(\ell M_*)^{-2} H|_{T=T_D} T_D^{-1} \\ &= f(M_5/M_*)^2 (M_5/M_4)^{11/2}. \end{aligned} \quad (53)$$

Since we should require $\ell^{-1} > \text{TeV}$, a constraint on M_5 becomes $M_5 > 10^{41/3} \text{GeV}$. The decoupling temperature becomes $T_D \sim 10^{11} (M_5/10^{41/3} \text{GeV})^{3/2} \text{GeV}$. If $M_* \sim \ell^{-1}$, then, we see $n_B/s \sim 10^{-10} (f/0.01) (M_5/10^{41/3} \text{GeV})^{3/2}$. To obtain the reasonable value of n_B/s , we must set M_5 to be around 10^{14}GeV . This is consistent with the tabletop experimental lower bound $M_5 > 10^8 \text{GeV}$ which is required on the positive tension brane.

V. SUMMARY

In this paper we considered the baryogenesis in the Randall-Sundrum type model with the bulk complex

scalar field. We found the baryon number violating process via the spacetime dynamics on the brane. Even if the scalar field do not have the potential, this mechanism can work. By assuming an appropriate source of CP violation, a reasonable net baryon number density can be obtained. However, the model that we considered here is only toy models. A study in a realistic and phenomenological model is needed.

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 - [16] The baryon number escaped from the brane will back again to the same brane in Randall-Sundrum two brane system because the extra dimension is compactified. However, the point is the presence of the baryon number violating process on the brane. The details of this process is not important. C/CP violation is more important for the baryon number left in the present universe.